

# Coulomb corrections to low energy antiproton annihilation cross sections on protons and nuclei.

A. Bianconi<sup>1,2)</sup>, G. Bonomi<sup>1,2)</sup>, E. Lodi Rizzini<sup>1,2)</sup>, L. Venturelli<sup>1,2)</sup>, A. Zenoni<sup>1,2)</sup>

<sup>1)</sup>*Dipartimento di Chimica e Fisica per l'Ingegneria e per i Materiali, Università di Brescia, via Valotti 9, 25123 Brescia, Italy*

<sup>2)</sup>*Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy*

## A b s t r a c t

We calculate, in a systematic way, the enhancement effect on  $\bar{p}p$  and  $\bar{p}A$  annihilation cross sections at low energy due to the initial state electrostatic interaction between the projectile and the target nucleus. This calculation is aimed at future comparisons between  $\bar{n}$  and  $\bar{p}$  annihilation rates on different targets, for the extraction of pure isospin channels.

# 1 Introduction

Recently, several sets of new data about antinucleon annihilation on nucleons and nuclei at very low energy have become available[1, 2, 3, 4, 5, 6, 7], and further measurements could be performed in the next years[8]. Whenever a comparison between targets or projectiles with different electric charge is required, for better understanding the underlying strong interaction effects (e.g. for isolating pure isospin contributions), it is necessary to be able to subtract Coulomb effects. The aim of this work is to calculate, as precisely and univoquely as possible, Coulomb effects as functions of the target mass and charge numbers  $A, Z$ , and of the projectile momentum  $k$  in the range 30–400 MeV/c. We define  $R_{A,Z}(k) = \sigma_{charged}/\sigma_{neutral}$  as the ratio between  $\bar{p}$ -nucleus annihilation cross sections calculated including or excluding Coulomb effects, at the projectile momentum  $k$  (MeV/c) in the laboratory frame.

In *qualitative* sense the action of Coulomb effects in  $\bar{p}p$  annihilations is a well understood process[9, 10]. In a semiclassical interpretation, the electrostatic attraction acts as a focusing device, which deflects  $\bar{p}$  trajectories towards the annihilation region. In quantum sense we may simply say that it increases the relative probability for  $\bar{p}$  to be in the annihilation region. An estimation of this effect is possible by assuming that the actual annihilation center is pointlike and that there is complete independence, or factorization, between the effects of strong and Coulomb forces. Then  $R_{A,Z} \approx |\Psi_Z(0)/\Psi_o(0)|^2$ , where  $\Psi_o(\vec{r})$  is the function describing the free motion of a charge zero projectile, and  $\Psi_Z(\vec{r})$  describes the motion of  $\bar{p}$  in the Coulomb field of a pointlike central charge  $+Ze$ . In this approximation  $R_{A,Z} = 2\pi\lambda[1 - \exp(-2\pi\lambda)]^{-1}$ , with  $\lambda = Ze^2/\hbar\beta$  ( $\beta$  is the relative velocity and  $R_{A,Z}$  only depends on  $A$  via c.m. motion within this approximation), and becomes  $R_{A,Z} \approx 2\pi\lambda$  for small velocities. Usually, at small velocities, the cross sections for esoenergetic reactions between neutral particles follow the  $1/\beta$  law, which means constant frequency of annihilation events. The velocity comes in when the annihilation rate is divided by the incoming flux (perhaps suggesting that the cross section is not the most useful observable at very low energies). In the case of opposite charges for the particles

in the initial state, the above approximation suggests a  $Z/\beta^2$  law, at least at small  $\beta$ . However there are some limitations:

- (1) The experiments which are of interest for us cover a range of momenta (30–400 MeV/c) where velocities are not always small.
- (2) Proton and nuclear charges are not pointlike.
- (3) Some interplay may exist between the strong central potential and the action of the Coulomb forces that breaks the factorization of the two effects.
- (4) Some lower cutoff (in the momentum scale) must exist due to the action of the electron screening.

Concerning the last point, we have attempted some calculation with a modified version of the codes used for the rest of this work. The modifications were such as to take into account the electron screening, with Thomas-Fermi distributions, for heavy nuclei. As far as we trust the modified codes, we don't see relevant screening effects at momenta  $\approx 10$  MeV/c. Apparently the code outputs are stable and reliable, at least at these kinematics and for large nuclei. Nevertheless the need to have our codes covering with precision very different space scales (atomic and nuclear, with a difference of many orders of magnitude) suggests a certain care. E.g., we don't get reliable results for larger momenta or very light nuclei (small variations of the parameters produce unstable results). So we will postpone a discussion on this point to the time when we have some alternative cross checks of these screening effects. Magnitude considerations anyway suggest that they should not be relevant at 30 MeV/c. In heavy nuclei the Thomas-Fermi approximation[9] suggests a distance  $r_B/Z^{1/3}$  between the nucleus and the bulk of the electronic cloud surrounding it, which is much larger than  $1/(30 \text{ MeV/c}) \approx 6 \text{ fm}$  also for  $Z = 100$ .

As we can see later (see e.g. figs. 1 and 2) the limitations (1) (2) and (3) are effective, and our results show large disagreements with the above  $Z/\beta^2$  law, especially with heavy nuclei.

In our calculations, the electrostatic potential has been produced by a uniform charge distribution with radius  $1.25 A^{1/3} \text{ fm}$ . The annihilation is reproduced by an optical potential of Woods-Saxon form. For all but the lightest nuclei we have chosen zero real

part, radius  $1.3 A^{1/3}$  fm, diffuseness 0.5 fm, and strength 25 MeV for the imaginary part. We will name this potential “standard nuclear potential” (SNP). The calculations have been repeated after changing the optical model parameters, to check for dependence of Coulomb corrections on these parameters (more details are given in the next sections). For the cases of Hydrogen, Deuteron and  $^4\text{He}$  targets, where low energy data are available[1, 2, 3, 4], we have compared the results of the SNP with the outcome of more specific (and rather different) potentials, which better fit the data.

The two reasons that are behind the parameters of the SNP are that (i) its radius and diffuseness are consistent with the  $A$ -systematic parameters of the nuclear density[11], and (ii) for  $A = 1$  this potential reproduces very well the  $\bar{p}p$  annihilation data in all of the range 30–400 MeV/c[12]. Many other choices with and without a real part (both attracting and repulsive) or with different shapes can reproduce the same  $\bar{p}p$  data (an example is given below), however a direct generalization of many among these potentials to nuclear targets is not so easy.

Our ideal aim would be to be able to produce a curve  $R_{A,Z}(k)$  which is independent on the specific potential used to simulate the strong interactions. For  $k > 20$  MeV/c this is possible with very good precision in light nuclei and within a 10 % uncertainty in heavy ones, as we show later on. Larger uncertainties are confined to the region of very small momenta ( $k < 20$  MeV/c).

The greatest source of interplay between the annihilation potential and the Coulomb interaction is the inversion mechanism at low energies. As widely discussed elsewhere[12, 13] and as seemingly measured[2], at very low energies it may happen that a modification of the features of the nuclear targets, which apparently should imply more effective annihilation properties, gets the opposite results. E.g.,  $\bar{p}p$  annihilation cross sections are larger than  $\bar{p}D$  and  $\bar{p}^4\text{He}$  ones at low energies. Moreover mechanisms (like Coulomb forces) that could be expected to enhance the reaction rates can loose effectiveness in presence of a very strong annihilation core. E.g., inversion is present for  $k < 200$  MeV/c in the potential used by Brückner *et al* [14] (we name this potential BP from now onwards) to fit elastic  $\bar{p}p$  data at  $k \approx 200$  MeV/c. The inversion property was not reported by these

authors because, at that time, annihilation data at lower momenta were not available, so they didn't perform calculations for the inversion region. With lesser adjustments of the parameters, their potential (imaginary part: strength 8000 MeV, radius 0.41 fm and diffuseness 0.2 fm; corresponding parameters for the attractive real part: 46 MeV, 1.89 fm and 0.2 fm) can reasonably fit the  $\bar{p}p$  annihilation data which have been measured in later years. However, it is easy to verify that any increase in the strength or radius of the imaginary part of their optical potential leads to a *decrease* in the corresponding annihilation cross section for  $k < 200$  MeV/c. In addition, putting the elastic part of this potential to zero leads to a twice as large elastic cross section. Unfortunately, since this potential (which has the advantage to reproduce elastic  $\bar{p}p$  data too) uses radius  $\approx 0.4$  fm (for the imaginary part) and 1.9 fm (for the real part), and diffuseness 0.2 fm (for both), its generalization to heavier nuclear targets is not straightforward. So with heavy nuclei we prefer to use the above SNP with standard nuclear density parameters. Although the inversion properties of the SNP are not so evident as in the BP case, also its outcomes are by far not  $A$ -linear at low momenta and this introduces a dependence of  $R_{A,Z}$  on the chosen parameters. Anyway, the results that we show in the next section suggest that strong model dependence is confined to  $k < 20$  MeV/c, even though the inversion mechanism is effective at larger momenta.

Center of mass corrections have been included in all calculations and they are particularly relevant for comparison between  $\bar{p}p$ ,  $\bar{p}D$  and  $\bar{p}^4\text{He}$  annihilations.

Another general remark, which has a certain importance, is that the Coulomb focusing effect acts on the atomic scale and it is relevant at momenta that are smaller than the typical nuclear momenta. The consequence is that, if one uses  $\bar{p}(D,p)X$  reactions to calculate the  $\bar{p}n$  annihilation cross sections, these cross sections are as much Coulomb affected as the  $\bar{p}p$  ones. This happens because the projectile is attracted by the deuteron charge more or less the same way, either it annihilates on the proton or on the neutron. So, while isospin invariance requires complete equality between  $\bar{p}n$  and  $\bar{n}p$  annihilation rates, in practice it will not be so, unless one is able to use free neutron targets or antiproton targets.

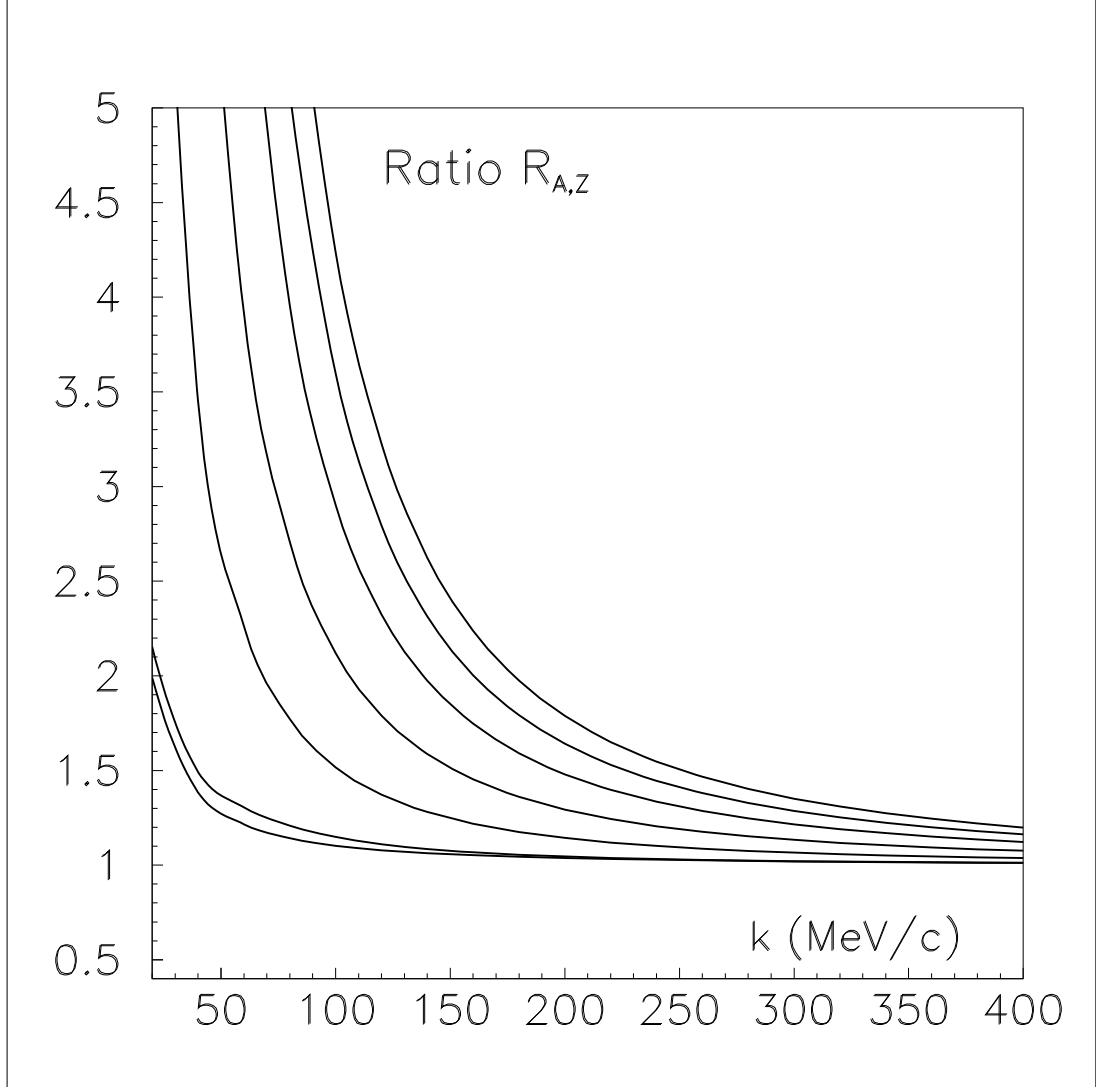


Figure 1: The ratio  $R_{A,Z}$  for target nuclei:  $H$ ,  ${}^4\text{He}$ ,  ${}^{20}\text{Ne}$  and then for  $A = 50, 100, 150, 200$  and  $Z = A/2$ . Upper curves correspond to increasing mass number. The small difference between the curves relative to Helium and Hydrogen is due to the compensation between charge and center of mass effects.

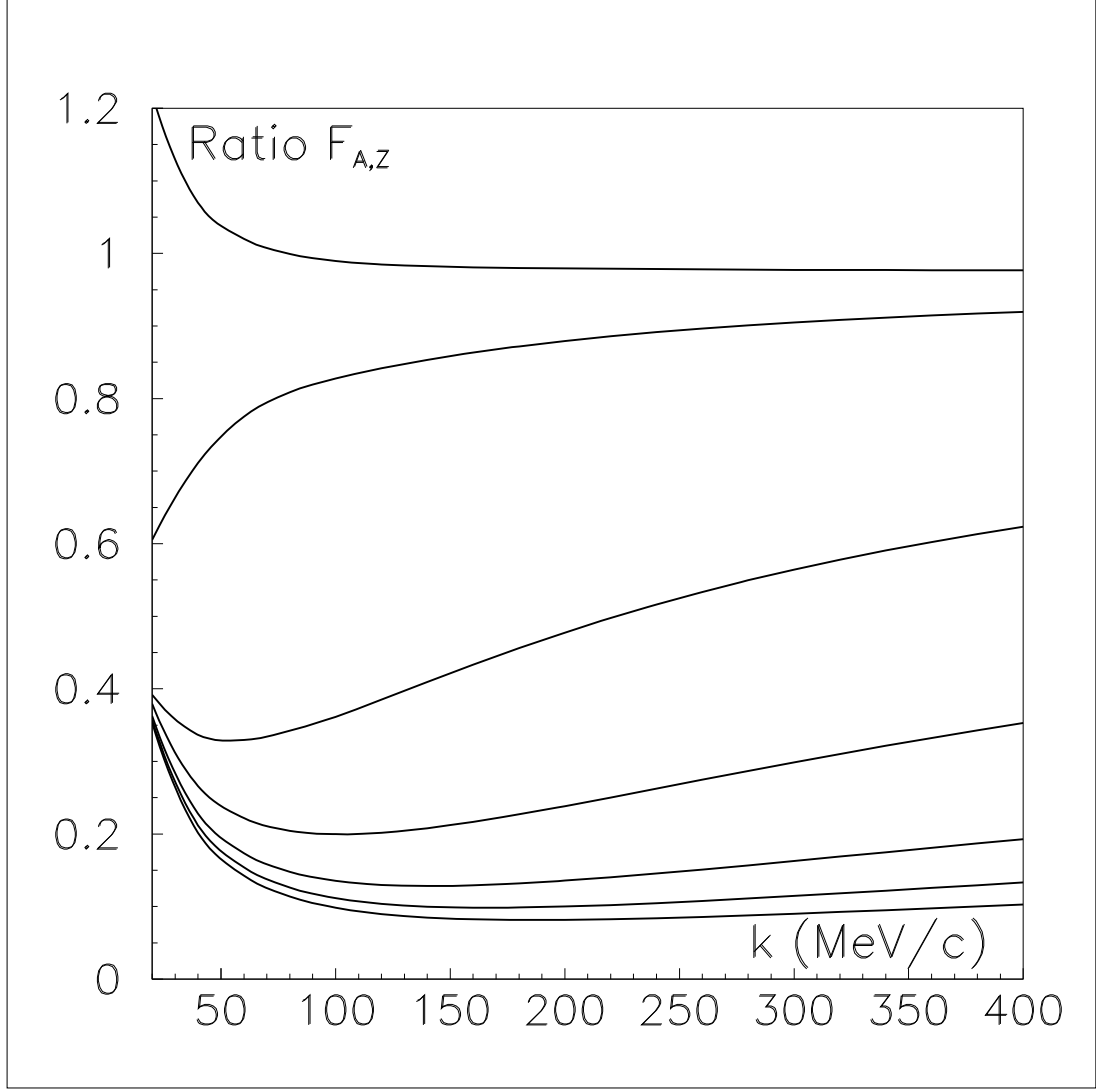


Figure 2: The ratio  $F_{A,Z}$  for target nuclei:  $H$ ,  ${}^4\text{He}$ ,  ${}^{20}\text{Ne}$  and then for  $A = 50, 100, 150, 200$  and  $Z = A/2$ . Lower curves correspond to increasing mass number. The Hydrogen curve reaches the value 1 at 80 MeV/c, and is  $\approx 0.98$  over 150 MeV/c. In the limit of respected pointlike prediction  $F_{A,Z}$  should be equal to 1, so  $Z \cdot F_{A,Z}$  expresses the “effective charge” of a nucleus.

## 2 Qualitative trends and dependence on the annihilation parameters

In fig.1 we show the ratio  $R_{A,Z}$ , calculated with the SNP, for targets H,  $^4\text{He}$ ,  $^{20}\text{Ne}$ , and for  $A = 50, 100, 150, 200$  and charge  $Z = A/2$ . It can give an idea, for each nuclear charge, of the momentum below which it is not possible to neglect Coulomb effects anymore.

Since  $R_{A,Z}$  changes by orders of magnitude at low momenta, a more reasonable quantity to be used to verify the dependence of  $R_{A,Z}$  on the annihilation parameters is the ratio  $F_{A,Z}$  between  $R_{A,Z}$  and its “pointlike” prediction  $R_{A,Z}^{(p)} = 2\pi\lambda[1 - \exp(-2\pi\lambda)]^{-1}$ , with  $\lambda = Ze^2/\hbar\beta$ . This ratio, shown in fig.2 for the same target nuclei of fig.1, is interesting in itself, because its deviations from  $F_{A,Z} = 1$  give an idea of the separation between the pointlike approximation and the actual nuclear behavior (notice: as we have tested, if one limits the pointlike approximation to the factor  $2\pi\lambda$  things change little). Not accidentally, the “pointlike” approximation is much worse in heavy nuclei. It is not so bad as far as the  $k$ -dependence is concerned, whereas it overestimates much the role of the nuclear charge. Indeed, in a wide range of momenta, we can write  $R_{A,Z} \propto Z_{eff}(k)/k$ , where the effective charge  $Z_{eff}(k)$  has a relatively slow dependence on  $k$  and becomes, at increasing  $A$ , much smaller than the real electric charge. The fact that with a proton or Helium target the pointlike approximation is good for  $k > 100$  MeV/c is of little relevance: as one can deduce by looking at fig.1, for light nuclei the charge has no role at these momenta.

A look at a log-log plot of the annihilation cross sections versus  $k$  with and without electric charge for heavy nuclei ( $A = 50, 100, 150$  and  $200$ , in fig.3) shows that the “neutral” cross section is the one that behaves in the most unpredictable way: it has a very small  $k$ -dependence for  $30 \text{ MeV/c} < k < 300 \text{ MeV/c}$ , and turns to a  $k^{-1}$  law at some  $k < 20 \text{ MeV/c}$ . In the region of  $k$ -independence these cross sections are roughly proportional to  $A^{2/3}$ , but become less  $A$ -dependent at decreasing momenta, in agreement with the described inversion. For  $k$  between  $100$  and  $300 \text{ MeV/c}$  the “charged” annihilation conforms to a rough  $k^{-1}$  law, and for smaller  $k$  to something like  $k^{-1.7}$  or  $k^{-1.8}$ . “Charged” and “neutral” cross sections are roughly proportional to a similar factor  $Z^\gamma$  or,



in other words,  $A^\gamma$ , with  $\gamma$  close to one. We notice that, if the charge were fully effective, the most obvious predictions, alternative to the optical potential model, would suggest a proportionality comprised between  $ZA^{1/3}$  and  $ZA^{2/3}$  for the “charged” cross sections, and between  $A^{1/3}$  and  $A^{2/3}$  for the “neutral” ones; the first law corresponds to the S-wave geometrical approximation  $\sigma_{ann} \sim R_{nucleus}/k$ , assuming imaginary scattering length  $\approx R_{nucleus}$ ; the second law is the Distorted Wave Impulse Approximation, where the nuclear cross section is more or less the sum of the cross sections of those nucleons lying on the nuclear surface. In all models, at  $k$  large enough, the charge effect should disappear.

With approximation 10 % (or slightly worse), for nuclei from intermediate to heavy we have found that it is possible to write  $\sigma_{ann}(\bar{p}A) \cdot \beta/Z \approx 10$  mb, for  $100 \text{ MeV/c} < k < 300 \text{ MeV/c}$ . For  $10 \text{ MeV/c} < k < 100 \text{ MeV/c}$  a corresponding law is  $\sigma_{ann}(\bar{p}A) \cdot \beta^\alpha/Z^{3/2} \approx 7$  mb, with  $\alpha = 1.7 \div 1.8$ . Of course, in these  $Z$  and  $Z^{3/2}$  dependences, charge and mass effects mix. In the fitting formulas of the next paragraph the roles of  $A$  and  $Z$  will be clearly separated (for the needs of the application to heavy nuclei with  $Z < A/2$ ).

Only at very low momenta  $\bar{p}-A$  annihilation cross sections follow the expected  $k^{-2}$  law. We have compared annihilations on nuclei with doubled momentum, i.e.  $k = 2 \text{ MeV/c}$  versus  $k = 4 \text{ MeV/c}$  and so on. With the  $k^{-2}$  law fully enforced, the corresponding ratio of annihilation cross sections should be 4. With Hydrogen target,  $\sigma_{ann}(k)/\sigma_{ann}(2k) = 4$  within four figures for  $k-2k = 1-2 \text{ MeV/c}$ , 3.65 for  $10-20 \text{ MeV/c}$ , 3.4 for  $15-30 \text{ MeV/c}$  and 3.2 for  $20-40 \text{ MeV/c}$ . Things are better with heavy nuclei: With  $A = 200$  and  $Z = 100$  we get 3.85 at  $20-40 \text{ MeV/c}$ .

This suggests that calculations of scattering lengths and similar low-energy quantities, based on the presently available annihilation data, and where Coulomb effects are subtracted via the pointlike approximation, should be at least compared with optical potential analogous calculations.

A last observation is that in fig.1 the ratio  $R_{A,Z}$  is almost identical, despite the charge difference, for Hydrogen and  $^4\text{He}$  targets. This is due to the compensation between center of mass momentum shift and Coulomb focusing. Not considering the center of mass transformation can lead to large errors in the interpretation of light nucleus data.

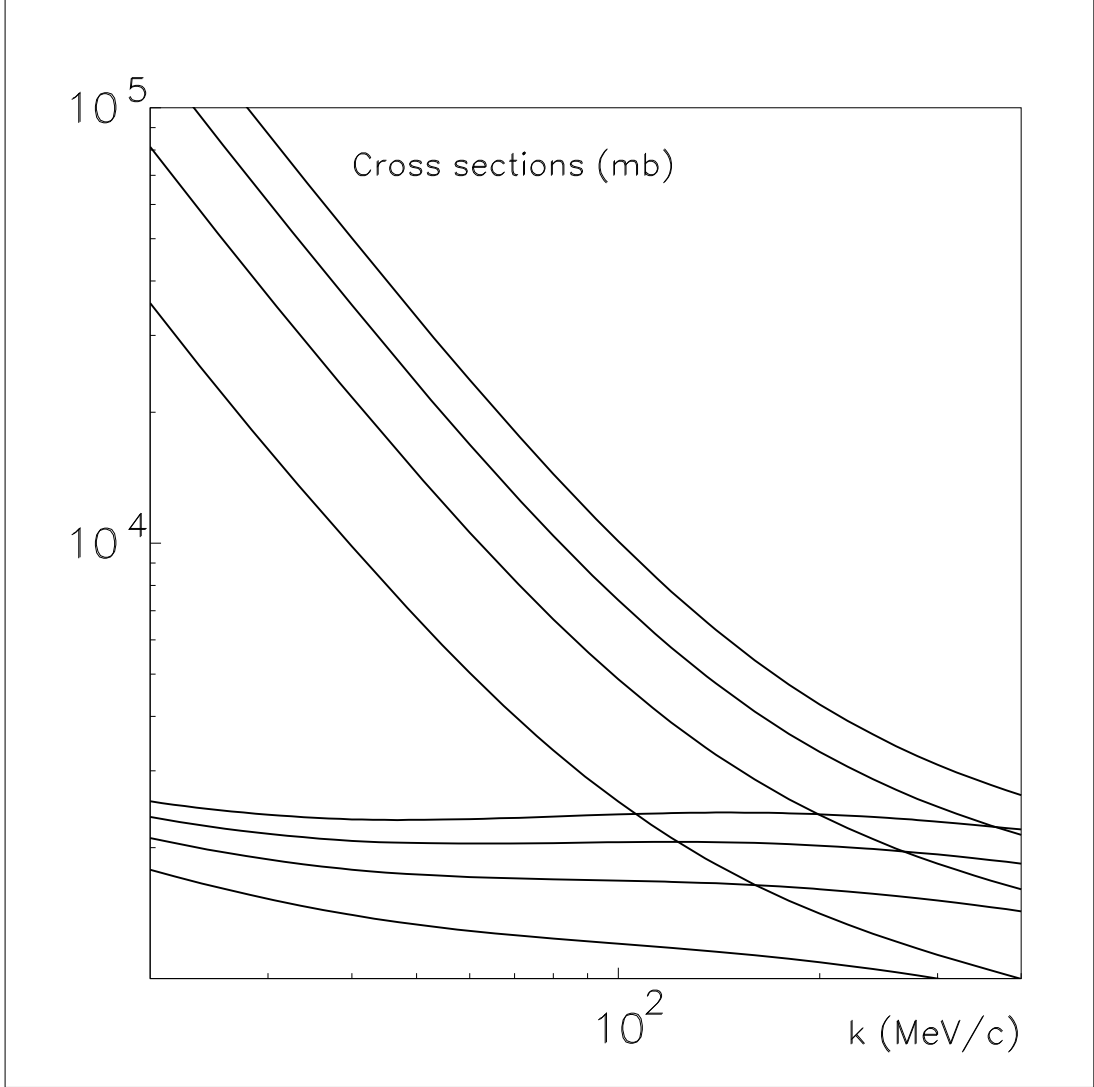


Figure 3: *The four upper curves represent  $\bar{p}A$  annihilation cross sections calculated within SNP including Coulomb effects, in the range of  $\bar{p}$  momenta from 20 to 400 MeV/c. The lower curves reproduce the same, but without Coulomb effects. In both cases, larger cross sections correspond to increasing mass numbers. As well known, straight lines in log-log plots indicate power relations of the kind  $y = x^\alpha$ .*

Concerning the problem of the dependence of  $R_{A,Z}$  on the nuclear potential parameters, we must distinguish between the case of light and heavy nuclei. In the first case we have some low energy data that allow for preparing ad hoc potentials which, although perhaps artificially (e.g. via repulsive interactions), permit a reasonable reproduction of the available data. This allows us, with any specific light nuclear target, for a comparison between the outcome of the SNP and the outcome of a pretty different potential. This comparison does not show any relevant model dependence for  $R_{A,Z}$ , as showed in detail in the following. With heavy nuclei we have no alternatives to the SNP, so that comparisons have been performed by simply attempting some changes in the SNP strength and diffuseness and comparing the outputs. With this procedure, we can estimate a model dependence of  $R_{A,Z}$  below 10 % for heavy nuclei at momenta around 30 MeV/c, half of it at 50 MeV/c and a satisfactory model independence over 100 MeV/c.

This is clearly showed in fig.4, where we present  $F_{A,Z}$  for the cases  $A = 50$  and  $200$ ,  $Z = A/2$ , comparing the results for the three choices (i)  $W = 25$  MeV,  $a = 0.5$  fm, (ii)  $W = 12.5$  MeV,  $a = 0.5$  fm, (iii)  $W = 18$  MeV,  $a = 0.6$  fm. A variation of the optical potential by a factor two should include all the reasonable possibilities (much larger variations of the potential strength are allowed only jointly with compensating variations of the radius or of the diffuseness, which for heavy nuclei would not make too much sense). As one can clearly see in the figure, to stay safely within 10 %, one must select momenta from 30 MeV/c upward. Below 20 MeV/c the curves corresponding to different models seem to increase their separation in a less controllable way.

In general, one would need data to impose stricter constraints on the optical model parameters. In fact, a qualitative synthesis of many attempts with different potentials, in light and heavy nuclei (more details on light nuclei are presented in the next section), suggests that whenever two different potentials are such as to reproduce similar “charged” cross sections, also the corresponding “neutral” cross sections will be similar. So we can say that a certain value of annihilation cross section is associated with a certain  $R_{A,Z}$ , whatever potential has been chosen to reproduce this cross section.

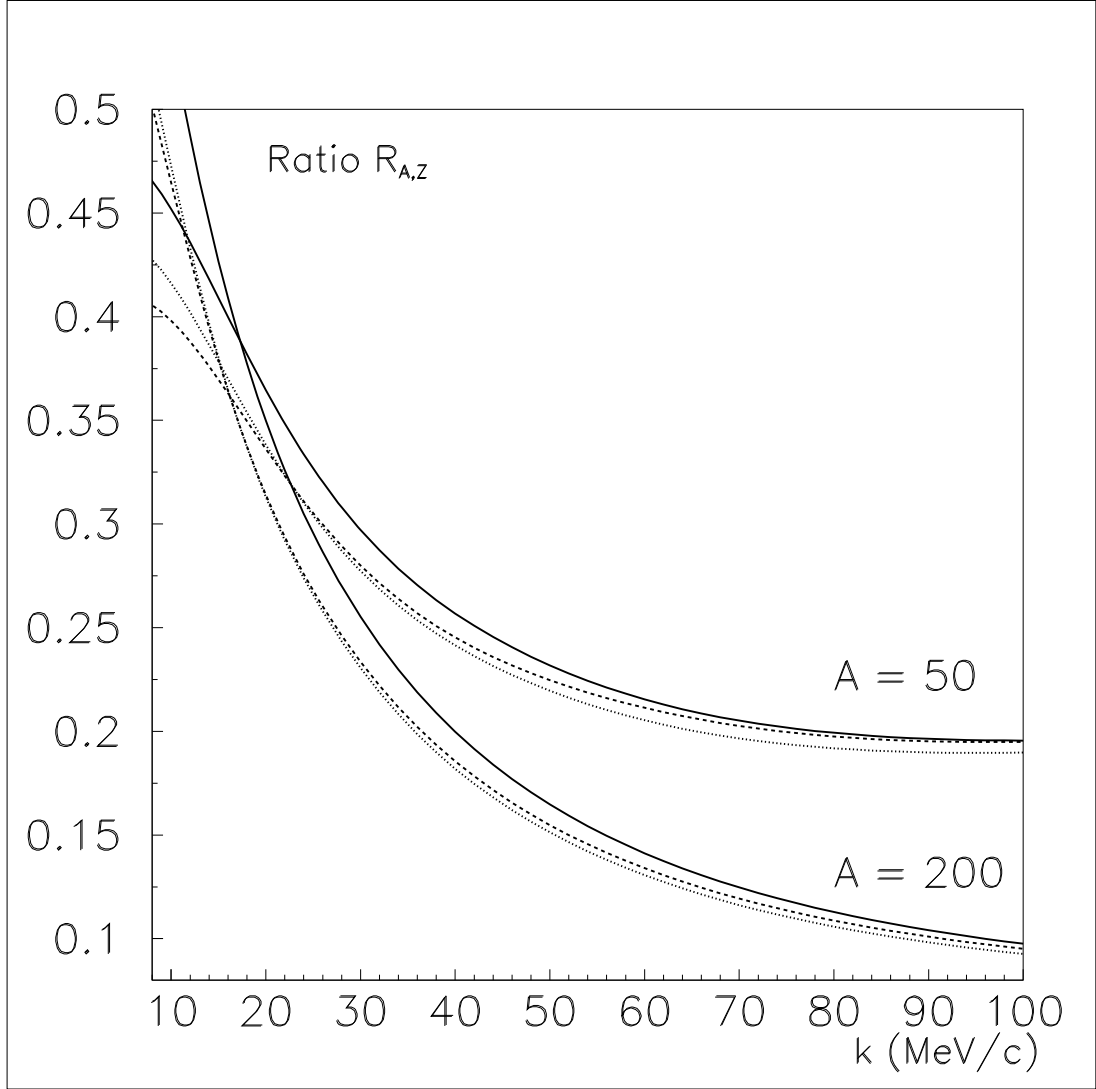


Figure 4: The ratio  $R_{A,Z}$  for target nuclei with  $A = 50$  and  $200$  and  $Z = A/2$ . For each nucleus we have used three different (pure imaginary and Woods-Saxon like) potentials, all with radius  $1.3 A^{1/3}$  fm and

- (i)  $W = 25$  MeV,  $a = 0.5$  fm (continuous line),
- (ii)  $W = 12.5$  MeV,  $a = 0.5$  fm (dashed line),
- (iii)  $W = 18$  MeV,  $a = 0.6$  fm (dotted line).

### 3 Fits of $R_{A,Z}$ .

In this section we synthesize the results of the calculation of  $R_{A,Z}$  on a wide spectrum of nuclei. We don't show figures, since these would simply report curves all very similar to the previous ones. We give analytical fits of these curves, which in subintervals of 30–400 MeV/c reproduce them within specified errors.

All the reported fits have the general form

$$R_{A,Z} = 1 + C_\alpha Z \beta_{cm}^{-\alpha}, \quad (1)$$

where  $C_\alpha$  is a constant coefficient, and  $\beta_{cm}$  is the relative velocity in the center of mass frame, calculated via the relativistic relations between center of mass momentum, energy and velocity for a projectile with reduced mass  $AM_p/(A+1)$ . Actually, there is some *small* difference between relativistic and nonrelativistic quantities at the larger momenta of the range only, so this precision is not necessary, and one may take  $\beta_{cm} = \beta_{lab}$ , since at nonrelativistic level the relative velocity does not depend on the reference frame.

With nuclei with  $A > 50$  the data for  $R_{A,Z}$  can be fit within a few percent, in the range of laboratory momenta 50–400 MeV/c, by the relation:

$$R_{A,Z} = 1 + 10^{-5}(45 - 0.0075A)Z\beta_{cm}^{-\alpha}, \quad A > 50, \quad 50 < k < 400 \text{ MeV/c}. \quad (2)$$

Choosing  $\alpha = 2.07$  one gets a precision of some percent in all of the range 50–400 MeV/c, whereas choosing  $\alpha = 2.08$  the fit becomes particularly precise in the region 100–400 MeV/c (in practice one does not distinguish the original and the fitted curve anymore), precise within 10 % at 70 MeV/c and within 20 % at 50 MeV/c.

The above fit gets worse with nuclei with  $A < 50$ . For  $A = 40$  it still gives a 10 % precision in the region 100–400 MeV/c (and a little worse for lower momenta). However a better fit (within 5 %) is:

$$R_{40,Z} = 1 + 0.00051Z\beta_{cm}^{-2}, \quad 50 < k < 400 \text{ MeV/c}. \quad (3)$$

Following the heavy nuclei rule, the coefficient of  $Z/\beta_{cm}^{2.07}$  would be 0.00041, instead of 0.00051. Actually the value 0.00051 is a compromise one. With 0.00052 there is

better precision (almost perfect superposition of curves) for  $k > 100$  MeV/c and 10 % overestimation at  $k = 50$  MeV/c. In the region 50–100 MeV alone a very good fit is:

$$R_{40,Z} = 1 + 0.00084Z\beta_{cm}^{-1.8}, \quad 50 < k < 100 \text{ MeV/c.} \quad (4)$$

For the relevant lighter nuclei, precise fits can only be obtained, as in the previous case, by systematically splitting the momentum range into two parts: 50–100 MeV/c and 100–400 MeV/c. We use the same formulas, with the same exponents and different coefficients. If we call  $C_2$  and  $C_{1.8}$  the coefficients of the  $Z\beta^{-2}$  and  $Z\beta^{-1.8}$  terms, we get:

For  $A = 20$ ,  $C_2 = 0.00066$  (which, apart from almost perfectly reproducing the range 100–400 MeV/c, gives a 12 % overestimation at 50 MeV/c) and  $C_{1.8} = 0.97$ . It is nevertheless possible a fit of all the range within a few percent with  $1 + 0.00088Z\beta^{-1.83}$ .

For  $A = 12$  we don't find a unique satisfactory fit for all of the required range. For the split ranges we get:  $C_2 = 0.00080$ , and  $C_{1.8} = 0.0011$ . The two above coefficients allow for a precision within some percent.

The coefficients for all of the nuclei with  $A$  between 12 and 40 can be interpolated quadratically exploiting the values given for the cases  $A = 12, 20, 40$ . This procedure should not introduce bigger errors than those ones which are related with the model dependence.

Among the light targets, the most important case is Hydrogen, for which we need Coulomb corrections down to 35 MeV/c. In this case it is possible to calculate the corrections related with two completely different models for the central potential, i.e. the SNP and the BP. Both produce the same annihilation rates in all of the considered range.

In the first case all the range 35–400 MeV/c can be fitted within 5 % by

$$R_{1,1} = 1 + C_2\beta_{cm}^{-2}, \quad 35 < k < 400 \text{ MeV/c,} \quad (5)$$

with  $C_2 = 0.00030$ . However, in the subrange 70–400 MeV/c the  $\beta^{-2}$  law can be more precise, with  $C_2 = 0.0040$ . This allows for an error within 1 % from 70 to 400 MeV/c, 2 % at 60 MeV/c and 5 % at 50 MeV/c. For the subrange 30–70 MeV/c an almost perfect fit is given by the law

$$R_{1,1} = 1 + C_{1.4}\beta_{cm}^{-1.4}, \quad 30 < k < 70 \text{ MeV/c,} \quad (6)$$

with  $C_{1.4} = 0.0120$ .

When one repeats the same fitting procedure starting with the BP for the annihilation core, differences are small. In the range 70–400 MeV/c the previous  $\beta^{-2}$  law is as good as in the previous case, with exactly the same coefficient  $C_2 = 0.0040$ . For the range 30–70 MeV/c the  $\beta^{-1.4}$  law is still very good, with a small modification in  $C_{1.4}$ . With the previous coefficient one gets an almost uniform 1-2 % overestimation of  $R_{1,1}$ . In this case the best coefficient is  $C_{1.4} = 0.0110$ . A fit within 5 % of the full range 35–400 MeV/c is possible by the  $\beta^{-2}$  form with  $C_2 = 0.028$  (instead of the previous 0.30).

It must be noticed that the fact that the calculated value of  $R_{1,1}$  is almost the same with two such different potentials makes this result quite reliable. The output of the two in terms of total reaction cross sections is the same, but their geometrical properties are completely different.

With  $^4\text{He}$  and D targets the understanding of the structure of the annihilation potential is not very good yet, both because of controversial interpretation of data (which show strong inversion properties) and because it is here impossible to rely on systematical nuclear properties. As in the H case we compare fits to the outcomes of two different potentials.

With  $^4\text{He}$  we have data starting from 45 MeV/c. We first calculate  $R_{4,2}$  with the SNP, that produces annihilation curves that don't pass too close to the two data points at 45 MeV/c and 70 MeV/c. Then we even try with a peculiar potential which, due to a slightly different annihilation core and to a repulsive elastic force, produces a better fit to the experimental data in the full range 45–400 MeV/c. The exact values are: imaginary strength 40 MeV, real (repulsive) strength 28 MeV, radius  $1.1 \cdot 4^{1/3}$  fm, diffuseness 0.7 fm (radius and diffuseness are equal for the real and imaginary parts).

In the case of the SNP, the range 80–400 MeV/c can be fitted with very good precision by the  $Z\beta^{-2}$  law with  $C_2 = 0.00130$ . The  $Z\beta^{-1.4}$  law allows for a rather good fit (within 3 %) in all of the range 40–400 MeV/c, with  $C_{1.4} = 0.0040$ . An improvement of the fit (to 1 %) in the region 40–100 MeV/c can be obtained by the law  $Z\beta^{-1.25}$  with  $C_{1.25} = 0.0070$ .

With the second kind of potential the above  $Z\beta^{1.4}$  fit even improves a little its accuracy (errors within 2 % in all of the range 40–400 MeV/c). No relevant differences are found between the  $R_{4,2}$  calculated via the two potentials.

With a Deuteron target we, again, apply different choices for the potential. First the nuclear standard (which in the deuteron case is surely not adherent to the physical situation) and then a completely different one (which better reproduces the lowest energy  $\bar{p}$ –deuteron data) with imaginary strength 750 MeV, repulsive real strength 400 MeV, real and imaginary radius 0.1 fm, real and imaginary diffuseness 0.6 fm. In practice the latter one is an exponentially decaying potential, having radius much smaller than diffuseness.

With the SNP we can perfectly fit  $R_{2,1}$  in the range 30–200 MeV/c with the  $\beta^{-1.4}$  law, with  $C_{1.4} = 0.0060$ . The same fit can be extended to the region 200–400 MeV/c with error within 1 %. In the latter region a better fit coefficient would be 0.0040 (this does not make a relevant difference, but with this coefficient the fitting law can be extended to much larger momenta). With the other potential, nothing changes. To be more precise, the calculated cross sections at  $k < 200$  MeV/c are rather different at momenta below 200 MeV/c, but  $R_{2,1}$  is the same in both cases.

The above comparisons between couples of pretty different potentials confirm that for light nuclei the calculation of  $R_{A,Z}$  is, at all practical purposes, model independent.

## 4 Summary and conclusions

To summarize, in the full range 30–400 MeV/c we are not able to give a simple and general law for the Coulomb corrections, of the kind of the one derived from the approximation of a pointlike annihilation center. We have shown that such an approximation is rather poor in this momentum range. We have given analytical approximations, within reported errors, for the calculated values of the Coulomb correction with several relevant target nuclei: H, D,  $^4\text{He}$ , and then  $A = 12, 20, 40, 50, 100, 150, 200$ , and variable  $Z$ . By interpolation it should be possible to reproduce Coulomb corrections for most nuclear targets, starting from our formulas. These analytical approximations are all of the form



$R_{A,Z} = 1 + C_\alpha Z/\beta^\alpha$ , with  $\alpha$  ranging from 1.25 to 2.08 and  $C_\alpha \ll 1$ . For light nuclei (H to  $^4\text{He}$ ) they should be reliably model independent, while for heavier nuclei it is safer to assume a residual 10 % dependence on the details of the specific model used for describing the annihilation process.

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